Module 2

Nuclear Power – The Core

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EDTECH 512 (Summer 2014)

Module 2: Mass Defect – Making Nothing Out of Something

Upon Completion of this Module, you will be able to:

- STATE the forces present within the nucleus of an atom and arrange these forces in order of strength.
- DEFINE the term Mass Defect and CALCULATE the Mass Defect of an individual atom.
- CALCULATE the binding energy and the binding energy per nucleon of an individual atom.
- CALCULATE the amount of energy released when a fission event occurs.

A Simple Math Problem

Let's start this chapter off with a simple math problem. We are going to total up all the masses of the particles we discussed previously to determine the mass of a $^{235}_{92}U$ atom. To do this we're going to need some information. We're going to need to know the number of protons in the atom, the number of neutrons and, of course, the number of electrons. Determining the number of protons is easy. From the previous module, we know that the lower left hand number in the standard notation is the number of protons. As you recall determining the number of neutrons is then a matter of subtracting the number of protons from the upper left hand number which is the total number of nucleons. Doing this gives of values of 92 protons, 143 neutrons and 92 electrons. We're then going to need the atomic masses of each particle: 1.00727 AMU for a proton, 1.00866 AMU for a neutron and 0.00055 AMU for an electron. And just because I'm a nice guy, I'm also going to give you the answer. The mass of a $^{235}_{92}U$ atom is 235.044 AMU. The math in the box below shows the calculation.

Calculate the Mass of a $^{235}_{92}U$ Atom

Mass of Protons = 92 * 1.00727 = 92.6688 AMU		92.6688 AMU
Mass of Neutrons = 143 * 1.00866 = 144.2384 AMU		+ 144.2384 AMU
Mass of Electrons = 92 * 0.00055 = 0.0506 AMU		+ 0.0506 AMU
	Total Mass =	236.9578 AMU??

Wait a minute! How can this be? I just told you that the mass of a $^{235}_{92}U$ atom was 235.044 AMU. It's not like a used Wikipedia as a source either. I got the information from an honest to goodness physics website. Where on earth did we go wrong?

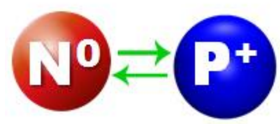
Forces Within the Nucleus

Before we can go about solving the mystery of the missing matter, we first have to examine the forces within the nucleus. There must be something holding the nucleus together, otherwise the universe would just be a series of protons and neutrons floating in space. It certainly would be much more random, but not nearly as useful.

The Gravitational Force

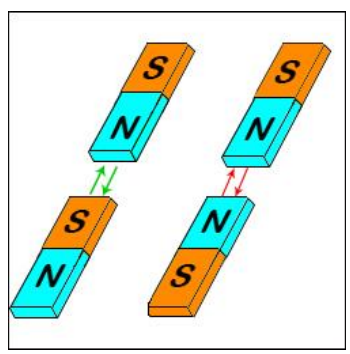
This is the force that we know and understand the best. It's the force that made the apple fall from the tree and knock poor old Issac Newton on the head. We understand gravity. It's all around us and it's the force that keeps us from flying out into space. Gravity is an attractive force between two objects which have mass. I have mass. You have mass. The Earth has mass, and protons and neutrons have mass. Therefore, there is a gravitational force between all nucleons. They attract each other. The equation for the gravitational attractive force between objects is

$$F = G * \frac{m_1 * m_2}{d^2}$$



where G is the gravitational constant, m_1 and m_2 are the mass of the objects and d is the distance between them. Obviously the distance between nucleons is incredibly small and under normal circumstances, the size of this denominator would result in a large value for the gravitational force. However, in this case the numerator is equally small. As previously discussed the masses of protons and neutrons are so small that we had to invent units of measure for them. Because of their lack of mass the resulting gravitational attractive force is also very small. The illustration on the right represents the attractive force between nucleons. For the remainder of this module, green arrows will represent attractive forces while red arrows will represent repulsive forces.

The Electrostatic Force



This is another force that we all intrinsically understand. It's the force you feel when you place two magnets next to each other. Magnets have two separate poles, a positive pole and a negative pole. If you place the two opposite poles close to each other, you feel the magnets try and come together. If you attempt to push two similar poles together, the magnets repel. The figure on the right represents what you feel.

The formula for the electrostatic force is very similar to the formula for the force of gravity.

$$F_e = k_e * \frac{q_1 * q_2}{r^2}$$

This time, however, rather than mass, the q represents the charge present on the particle. You recall from the last module that charge is one of the intrinsic properties of matter. The k is, of course a constant and we use r for radius instead of d for distance, but the equations are incredibly similar. This time, however, values matter. The values of q_1 and q_2 are enormous. How enormous, you ask? Well consider the distance between the nucleus and the surrounding electrons. As we said earlier, if one were to put a nucleus on one goal line of a football field, the outer electrons could be found on the opposite goal line. That's quite a distance, and yet the values of q_1 and q_2 within the atom are sufficiently large when compared to the distance that they work to keep the electrons in orbit. The charge values of these particles are significant, which of course raises the question, "If the positive charges on each proton is that significant, how do these protons gather together to form elements. There has to be another force working to hold them together.

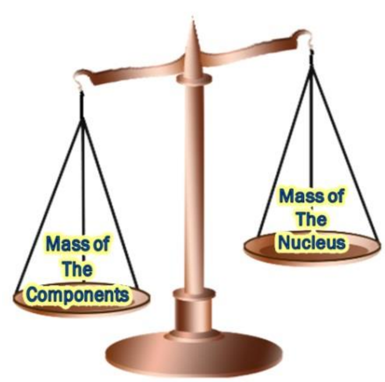
The Nuclear Force

There is another force present within the nucleus of an atom and that force is known as the nuclear force. The nuclear force is a strong force that only acts over very small distances between all nucleons within an atom. Think of the nuclear force as gravity on eleven. It's equally strong between protons and protons, neutrons and protons and neutrons and neutrons. The nuclear force is what holds everything in the universe together.

Of course holding the universe together comes at a cost. It's hard work keeping the universe from spinning out of control and the nuclear force comes at a price.

Everyone has heard of Albert Einstein. He was an incredibly smart guy, and if you ask anyone about Einstein's (except for particle physicists of course) they will invariably answer, "$E = mc^2$." I suppose now it would be a bit depressing for Mr. Einstein to have his life's work reduced to a three letter equation, but in fact Einstein had it right. That single equation is what holds the universe together and what allows us to generate thermal energy from nuclear fission.

While almost everyone in the world can quote the equation, a great many fewer know exactly what it means. Count yourself into the club because we're going to discuss it. The literal translation of the equation is this. Energy is equal to the mass of matter times the speed of light squared. So what does it really mean for us? Quite simply it means that the nuclear force takes a bit of mass away from the protons and the neutrons within a nucleus and uses that mass to create the nuclear force to hold the nucleons together. The difference between the mass of the individual nucleons and the nucleons within the nucleus is known as the mass defect, and the energy required to keep the nucleus together is referred to as the binding energy of the atom.



Mass Defect

Size Does Matter

When we're referring to the nucleus of an atom, size does matter. This is because the electrostatic force we mentioned earlier is a cumulative force. That is to say that the more positive charges there are within a nucleus, the more that nucleus wants to break apart. Therefore, the nuclear force within that nucleus has to be that much greater in order to hold it together. As we've seen with Einstein's equation, a greater force will require a greater mass and therefore the mass defect will be larger, but suppose we were to take a single large atom and split it into two smaller atoms. What would happen then?

Before we tackle that question, we first need to be able to work with the Einstein equation. Let's take a look at the example we started with, a single $^{235}_{92}U$ atom. From our calculations on the first page of the module text, we know that the mass of the components of a $^{235}_{92}U$ atom is 236.9578 AMU. From the answer I gave you, we know that the actual mass of a $^{235}_{92}U$ is 235.044 AMU. I should mention here that you will have to push the "I Believe" button. While I certainly could go through the conversion and derivation of the constant I am going to give you, I've no doubt that your eyes would glaze over and you would start shaking uncontrollably. Conversions are necessary, wonderful things, but they're beyond the scope of this course. Suffice to say, for the purposes of this course, 1 Atomic Mass Unit of matter is equivalent to 931.5 Mega-electron Volts. For those of you who are wondering, a Mega-electron Volt (MeV) is a unit of energy, like a joule or a calorie. So let's see how much energy it takes to bind a $^{235}_{92}U$ together.

Nuclear Force Within a $^{235}_{92}U$ Nucleus

Energy(MeV) = Δ mass (AMU) * 931.5 MeV/AMU

Energy(MeV) = [236.9578AMU – 235.0440AMU] * 931.5 MeV/AMU

Energy(MeV) = 1.9138 AMU * 931.5 MeV/AMU

Energy(MeV) = 1782.7 MeV

The above calculation shows that the force required to hold a $^{235}_{92}U$ nucleus together is 1782.7 MeV. When that nucleus was formed, the protons and neutrons within it gave up enough mass each to provide the nuclear binding force. Pretty nice of them, don't you think? Well this is a great specific example, but as we will see further along, we're going to need to crunch these numbers over and over. We're going to need a generic formula and a means to calculate this force which works for all isotopes of any element. We're going to need a process.

Calculating the Binding Energy

The next series of equations use the standardized model but in its generic form. Please don't get wound around the axel because this is a generic equation. There's nothing to be afraid of. All of you already understand the material. We'll start with a generic atom whose chemical symbol is X

Given an isotope of X with the Standardized Notation Form $^{A}_{Z}X$ and an atomic mass of X equal to N AMU.

Mass Defect (Δm) AMU = { [Z * 1.00727 AMU] + [(A-Z) * 1.00866 AMU] } – N AMU

Binding Energy (E) MeV = Mass Defect (Δm) AMU * 931.5 MeV/AMU

When taken at face value the equations above only make sense. We first must find the weight of the components that make up the nucleus. To do this, we multiply the number of protons (Z) by the weight of a proton in AMU. Next, we must find the weight of all the neutrons in the nucleus. As we recall from Module 1, the number of neutrons is equal to the total number of nucleons (A) minus the number of protons (Z). Once we total up the weight of the components of the nucleus, finding the mass defect is simply a matter of subtracting what the atom weighs in reality from what the atom ought to way based on the makeup of its nucleus. Once the mass defect has been found, calculating the binding energy is a breeze. We simply multiply the mass defect by our given constant.

Introducing the Fission Process

Now that we know how to compute binding energy, let's answer the question we asked on the previous page. If you recall, we wondered what would happen if a large atom was split into two smaller atoms. The process by which we do this is called fission.

Fission occurs when a large atom absorbs an extra neutron. When this happens the nucleus becomes unstable and the atom splits into two smaller parts. We'll cover why this happens in a moment, but for right now, let's just accept that it happens. The two elements which emerge from a fission event are called fission products. These fission products vary according to a probability curve, but right now that's not important. We are going to look at a specific event. For the purposes of this discussion, we're going to imagine a $^{235}_{92}U$ atom absorbs a neutron and splits into $^{90}_{37}Rb$ and $^{143}_{55}Cs$ and three neutrons.

This is the nuclear equation that represents that event:

$$^{235}_{92}U + ^{1}_{0}n \rightarrow (^{236}_{92}U)^{*} \rightarrow ^{90}_{37}Rb + ^{143}_{55}Cs + (3)\ ^{1}_{0}n$$

The asterisks after the $^{236}_{92}U$ signifies that this nucleus is in an excited state and unstable. This excited state will lead to the splitting of the Uranium atom into a Rubidium atom and Cesium atom. So let's analyze this equation from a mass defect standpoint. We will first examine the mass on the far left side and compare it to the mass on the far right hand side.

The mass on the far left hand side is equal to the mass of a $^{235}_{92}U$ atom plus the mass of a neutron.

We know from our previous example that the mass of a $^{235}_{92}U$ is 235.0440 AMU.
We also know that the mass of a neutron is 1.00866 AMU.
Therefore: 235.0440 + 1.00866 = **236.0527 AMU**. (Mass of Left Side of Equation)

The mass on the far right hand side is equal to the mass of a $^{90}_{37}Rb$ atom plus the mass of a $^{143}_{55}Cs$ atom plus three neutrons.

By consulting a table, we know that the mass of $^{90}_{37}Rb$ is 89.915 AMU.
By consulting a table we also know that the mass of $^{143}_{55}Cs$ is 142.927 AMU.
Therefore: 89.915 + 142.927 + (3*1.00866) = **235.868AMU** (Mass of Right Side of Equation)

As you can plainly see, there is a mass disparity between the left and right hand sides of the equations. Since we know that mass and energy need to balance between left and right sides, we know that the missing mass was released as energy. They type of energy released is unimportant right now. It is enough to know that while we've lost mass in the system, we've gained energy. But how much energy have we gained from this particular event? Well luckily for us, Mr. Einstein's equation still holds true. We can use our binding energy equation to calculate the energy released from this fission event.

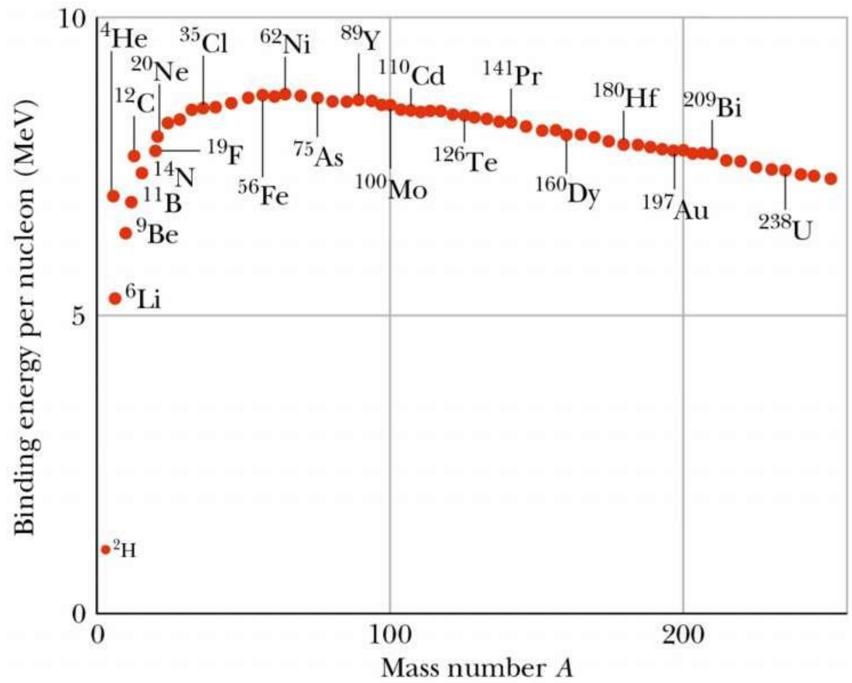
Mass Defect (Δm) AMU = 236.0527 AMU – 235.868 AMU = 0.1847 AMU
(E) MeV = Mass Defect (Δm) AMU * 931.5 MeV/AMU = **172.048 MeV**

Now 172 MeV may not seem like a lot of energy, but when you understand that trillions and trillions of fissions take place each second in a nuclear reactor, you begin to appreciate the difference a few trillion neutrons can make. There's one last topic to cover in this module and it concerns why Uranium fissions in the first place.

Did you know??

There are approximately 48,500,000,000,000,000,000,000 atoms in a single cubic centimeter of Uranium

Binding Energy per Nucleon



Take a look at this graph. It's a graph of the binding energy per nucleon for the atoms on the periodic table. What is binding energy per nucleon? Well it's just as the name suggest. Its value is found by dividing the binding energy of an atom by the number of nucleons in the nucleus, but what does the graph mean?

Think of it this way. Imagine you're holding ten tennis balls in both your hands. Your friend comes along and decides to try to pull one of the tennis balls free from your hands. To his surprise, however, you're very good at keeping the tennis balls where they are, but the effort is overwhelming and you're barely hanging on to all ten balls. Your friend sees that you're struggling and decides that he's going to make you drop all those tennis balls, no matter what. He knows he can't do it by brute strength because he has already tried so he does the next best thing. He gives you one other tennis ball and tells you that now you have to hold on to eleven balls instead of ten. The next time he tries to pull a ball free, the effort is too much for your and the balls go scattering all over the floor.

This is why Uranium is susceptible to fission. We've already said that the nuclear force is a strong force, but that it only acts over short distances. Therefore, the further away the nucleons get from the center of the nucleus, the weaker the nuclear force gets. As you can tell by the graph above, the nuclear force reaches its maximum at about 62 nucleons. After that, the nuclear force stays constant, but the number of nucleons increases so the binding energy per nucleon decreases and you drop your tennis balls. This is why the heavier elements are less stable and this is why Uranium makes a good nuclear fuel.

Looking Ahead

There are other ways that atoms change that don't involve fission. In the next module we'll look at radioactive decay and the effects that this decay has on the overall heat that's generated by a nuclear reactor. We'll also be looking at the Table of Nuclides and isotopes, isotopes, isotopes.